Selected exercises solved

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December 8, 2001

Solution of Exercise 2.29:

$$\mathsf{in} = \mathsf{hd}^{-1} \cup \mathsf{in} \circ \mathsf{bd}^{-1}, \qquad \mathsf{hd} \subseteq (\, \iota \cup \mathsf{in}^{-1} \setminus \mathsf{in}^{-1} \circ \mathsf{in}\,) \circ \mathbb{1}.$$

Solution of Exercise 2.30: $\mathsf{at}(\,X\,) \leftrightarrow \mathsf{hd}(\,X\,)\texttt{=}X \neq \texttt{[ ]}$, etc.

Solution of Exercise 2.31:

- When $\mathcal{F} = \emptyset$.
- Not always, because requiring that $\approx_{\mathcal{H}}$ be an equivalence relation does not suffice. The following two congruence conditions must also be met when $s \approx_{\mathcal{H}} d$:
  - $t[s] \approx_{\mathcal{H}} t[d]$ if t belongs to $\mathbb{H}(\mathcal{C}, \mathcal{F})$;
  - $\alpha[d]$ is one of the atomic formulae which have been selected as being true in the base, if $\alpha[s]$ is one of them.

Solution of Exercise 2.32: Let us take a model $\Im$ of $\mathsf{Cn}(I\!\!P)$ and a corresponding interpretation $\mathcal{H}$, according to Theorem 2.2. Since $\alpha^{\Im} = \mathbf{t}$ holds for every α in $I\!\!P$, one also has $\alpha^{\mathcal{H}} = \mathbf{t}$ by Theorem 2.2. Therefore $\vartheta^{\mathcal{H}} = t$ holds for every ϑ in $\mathsf{Cn}(I\!\!P)$. As we have observed already, the domain of $\mathcal{H}$ is countable.

Solution of Exercise 4.7: The desired algorithm is described by the SETL program pairsTuples.stl; but the complexity analysis is not carried out in the current version of these notes.

Solution of Exercise 5.18: The desired algorithm is described by the SETL program vennRegions.stl; but the complexity analysis is not carried out in the current version of these notes.